WEEK06. Linear Algebra

Kim, Hyun-Min

Department of Mathematics, Pusan National University

Polynomial Version

Linear System

solve:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{m1}x_1 + a_{m2}x_2 + \ \dots + a_{mn}x_n = b_m \end{cases}$$

aim :

$$\begin{cases} x_1 = s_1 \\ x_2 = s_2 \\ \dots \\ x_n = s_n \end{cases}$$

Possible operation

1. Exchanging two equations
2. Multiplying any nonzero constant
3. Multiplying any nonzero constant and add to another equation

Matrix Version

Linear System –〉 Matrix Version

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ \ldots + a_{2n}x_n = b_2 \\ \cdots \\ a_{m1}x_1 + a_{m2}x_2 + \ \ldots + a_{mn}x_n = b_m \end{cases} \rightarrow \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$ (matrix)

$$\rightarrow \begin{bmatrix} a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & \ddots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} & b_n \end{bmatrix}$$ → (using Possible operation)

$$\rightarrow \begin{bmatrix} \widetilde{a_{11}} & \cdots & \widetilde{a_{1n}} & \widetilde{b_1} \\ 0 & & \widetilde{a_{2n}} & \vdots \\ \vdots & \ddots & \vdots & \widetilde{b_n} \\ 0 & \cdots & \widetilde{a_{mn}} & \end{bmatrix}$$ (Upper triangular matrix)

(This is Gaussian elimination)

Determinant

$\mathrm{Ax} = \mathrm{b},\ \mathrm{A} \in \mathbb{R}^{n\times n}$

If $\det(\mathrm{A}) \neq 0$, then $\mathrm{x} = \mathrm{A}^{-1}\mathrm{b}$

$$\det(\mathrm{A}) = \mathrm{a}_{i1}(-1)^{i+1}\det(\mathrm{A}_{\mathrm{i1}}) + \mathrm{a}_{i2}(-1)^{i+2}\det(\mathrm{A}_{\mathrm{i2}}) + \cdots + \mathrm{a}_{in}(-1)^{i+n}\det(\mathrm{A}_{\mathrm{in}})$$

$$= \sum_{j=1}^{n} a_{ij}(-1)^{i+j}\det\left(\mathrm{A}_{\mathrm{ij}}\right)$$

Operation Counts of det

$$\begin{cases} n!\displaystyle\sum_{k=1}^{n-1}\frac{1}{k!} : \times \\ n! - 1 : + \end{cases}$$